

# How to obtain estimates of vertical viscosity from surface drifter data

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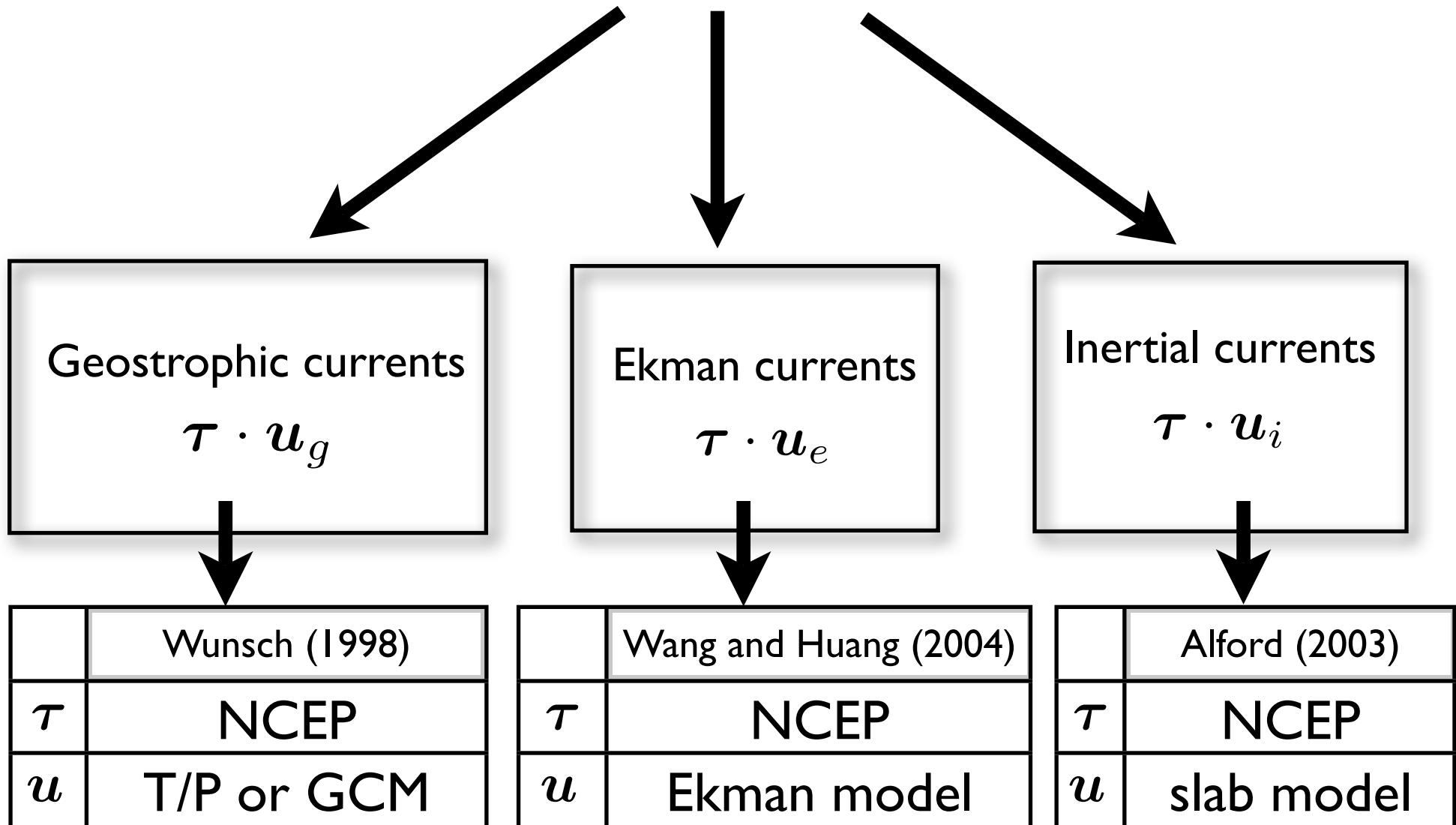


NASA/JPL NSF  
Peter Niiler, Glenn Ierley

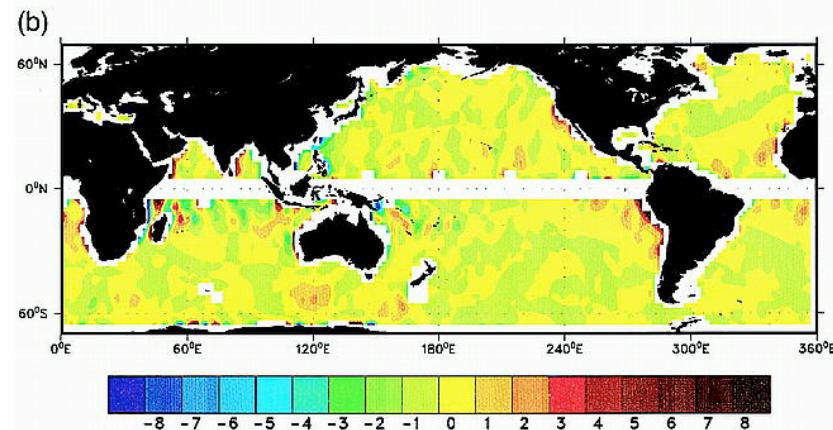
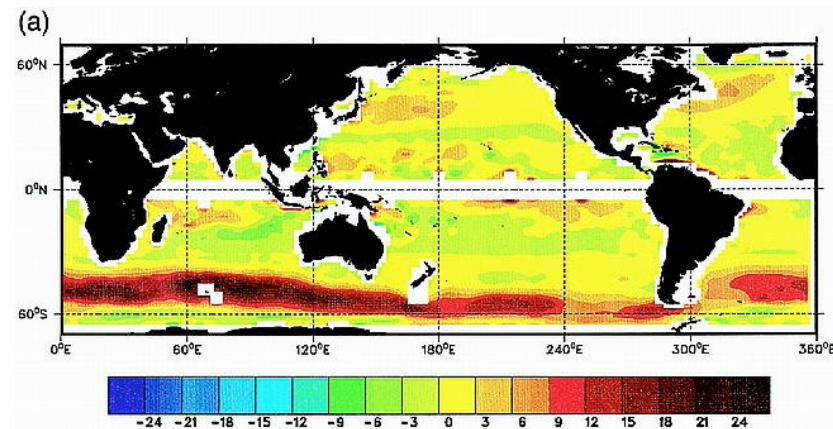
# Why?

- Need to quantify the momentum fluxes from the atmosphere to the ocean
- Need to identify the pathways of these fluxes

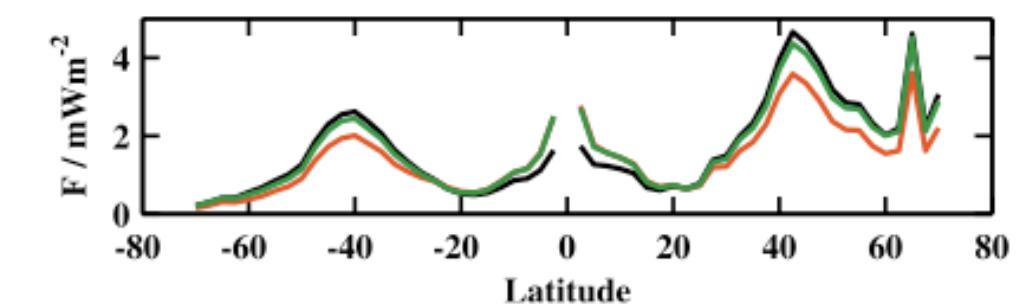
# Wind Energy Input Rate



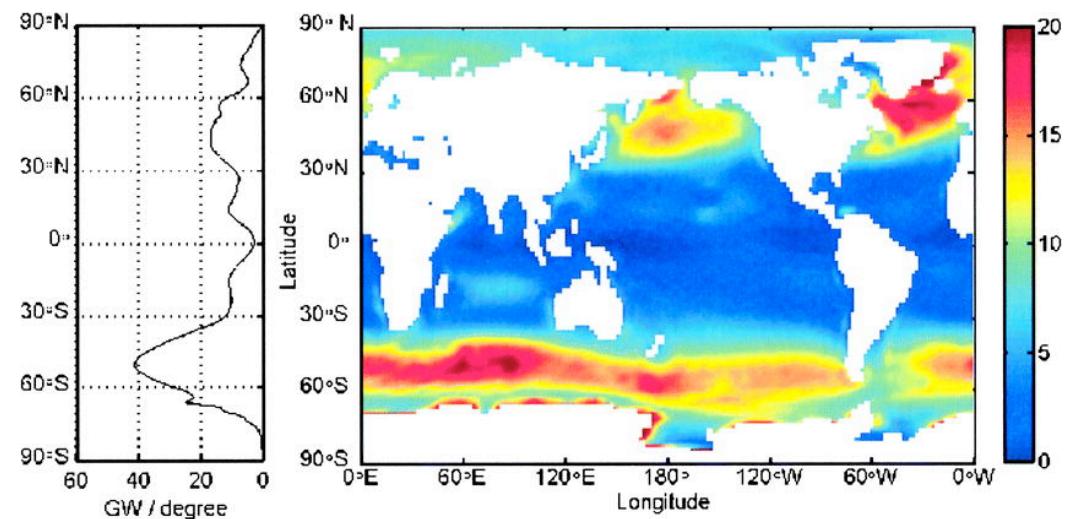
# Wunsch (1998)



# Alford (2003)



# Wang and Huang (2004)



# Outline

1. Vertical Viscosity in the Ekman layer  
Ekman models and transfer functions

Energy Input to the Ekman Layer  
Spectral Energy Equation

2. Estimates from surface drifter velocity data
3. Results in the Southern Ocean

# 1. Vertical viscosity in the Ekman Layer

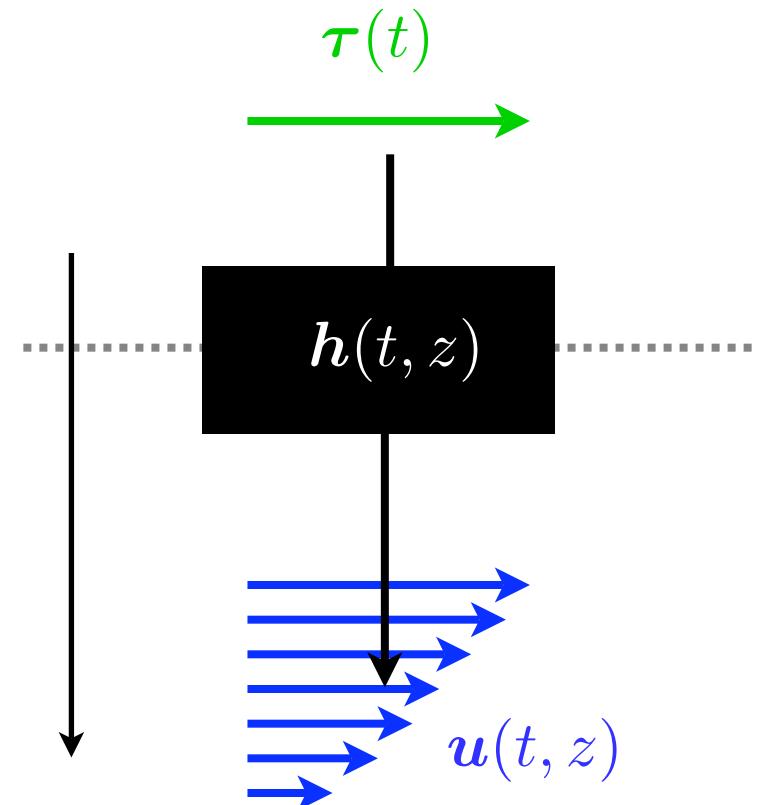
# The transfer function

$$u(t, z) = \int_0^\infty h(t', z) \tau(t - t') dt'$$

$$\int_{-\infty}^{+\infty} (\cdot) e^{-i2\pi\nu t} d\nu$$

$$U_\nu(z) = H_\nu(z) T_\nu$$

transfer function

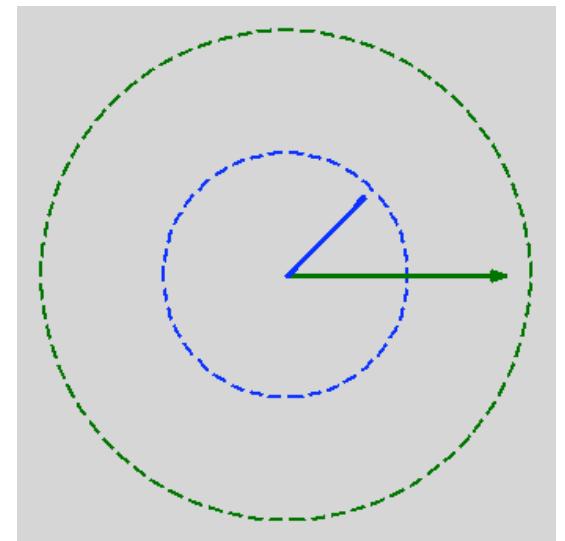
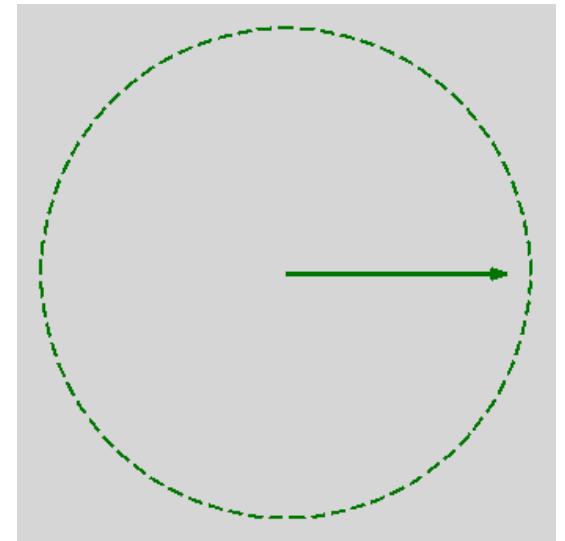


# The transfer function

monochromatic wind stress:

$$\mathbf{T}_\nu = 1 \times \delta(\nu - \nu_0) \rightarrow \boldsymbol{\tau}(t) = 1 \times \exp(+i2\pi\nu_0 t)$$

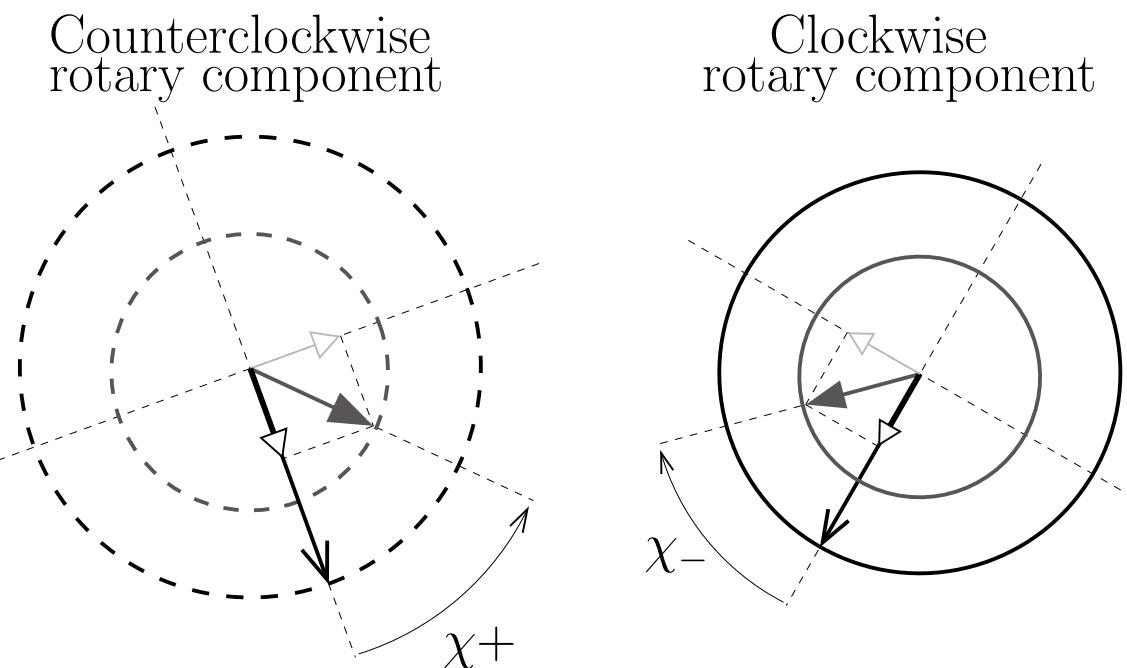
$$\begin{aligned}\mathbf{u}(t, z) &= \int_{-\infty}^{+\infty} \mathbf{U}_\nu(z) \exp(+i2\pi\nu t) d\nu \\ &= \int_{-\infty}^{+\infty} \mathbf{H}_\nu(z) \mathbf{T}_\nu \exp(+i2\pi\nu t) d\nu \\ &= \int_{-\infty}^{+\infty} \mathbf{H}_\nu(z) \delta(\nu - \nu_0) \exp(+i2\pi\nu t) d\nu \\ &= \mathbf{H}_{\nu_0}(z) \exp(+i2\pi\nu_0 t).\end{aligned}$$



# The transfer function

$$\mathbf{u}_{\nu_0}(t, z) = \mathbf{H}_{\nu_0}(z) \exp(i2\pi\nu_0 t) \quad \chi_{\nu_0}(z) = \tan^{-1} \frac{\mathcal{I}(\mathbf{H}_{\nu_0}(z))}{\mathcal{R}(\mathbf{H}_{\nu_0}(z))}$$

$$\mathbf{u}(t, z) = \sum_{-\infty}^{+\infty} \mathbf{u}_\nu(t, z)$$



→ wind stress  
→ ocean velocity { → crosswind component  
                  { → downwind component

# Ekman models

$$\frac{\partial \mathbf{u}(t, z)}{\partial t} + if\mathbf{u}(t, z) = \frac{1}{\rho} \frac{\partial \boldsymbol{\tau}(t, z)}{\partial z}$$

$$\frac{\boldsymbol{\tau}}{\rho} = -K(z) \frac{\partial \mathbf{u}(t, z)}{\partial z}$$

$$\int_{-\infty}^{+\infty} (.) e^{-i2\pi\nu t} d\nu$$

$$i(2\pi\nu + f)\mathbf{U}_\nu(z) - \frac{d}{dz} \left[ K(z) \frac{d\mathbf{U}_\nu(z)}{dz} \right] = 0$$

vertical viscosity

$K(z)$  vertical viscosity profile

$$-K(z) \frac{d\mathbf{U}_\nu(z)}{dz} = \frac{\mathbf{T}_\nu}{\rho}, \quad z = 0$$



$$\mathbf{U}_\nu(z) = \mathbf{H}_\nu(z) \mathbf{T}_\nu$$

$$\mathbf{H}_\nu(z) = \frac{\mathbf{U}_\nu(z)}{\mathbf{T}_\nu}$$

# Ekman models $K(z)$

constant  
 $K_0$

linear  
 $K_1 z$

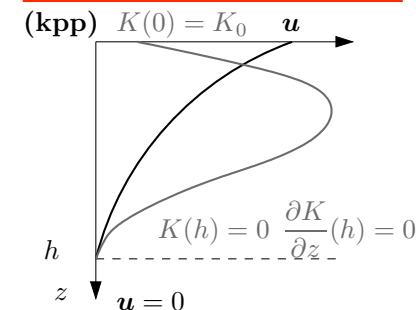
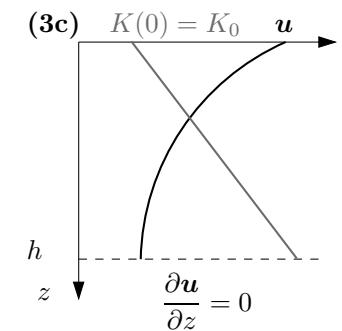
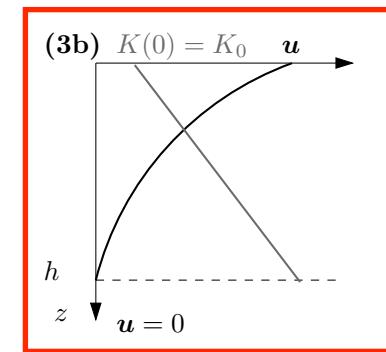
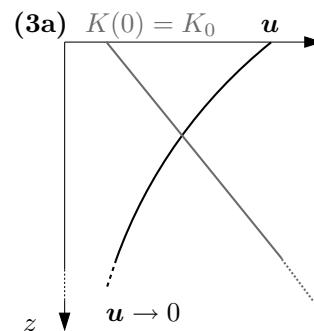
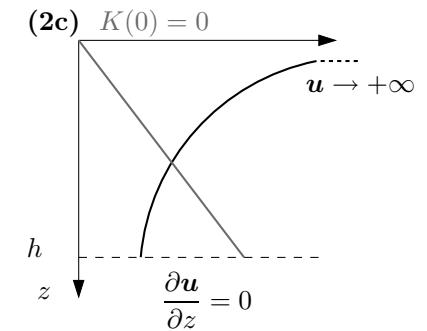
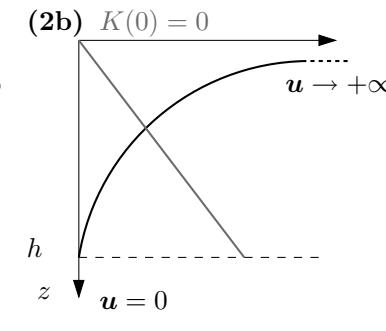
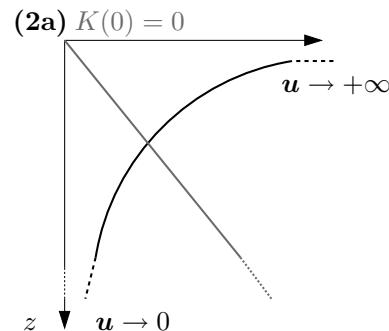
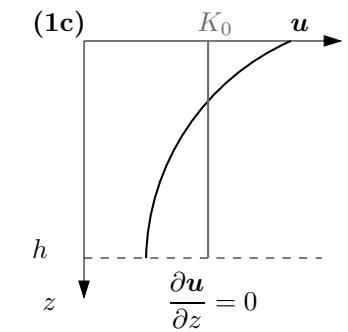
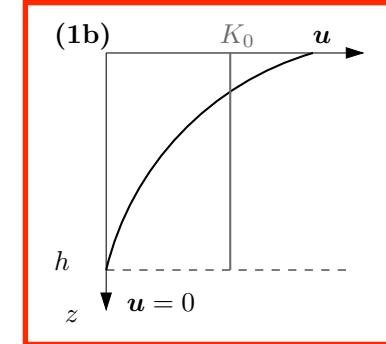
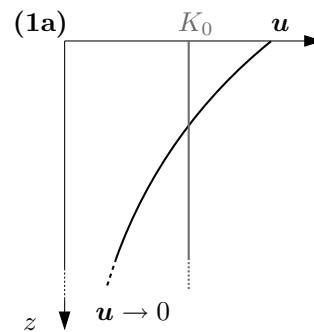
linear  
 $K_0 + K_1 z$

cubic

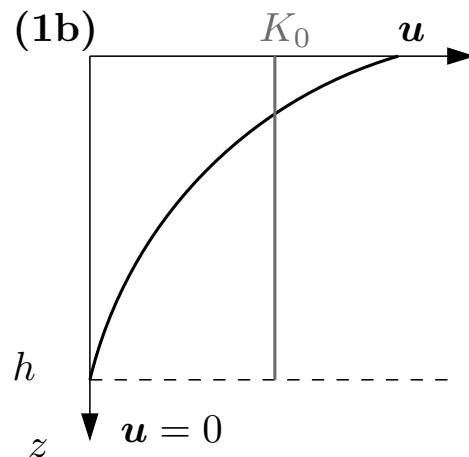
infinite

1 - layer

1 1/2 - layer

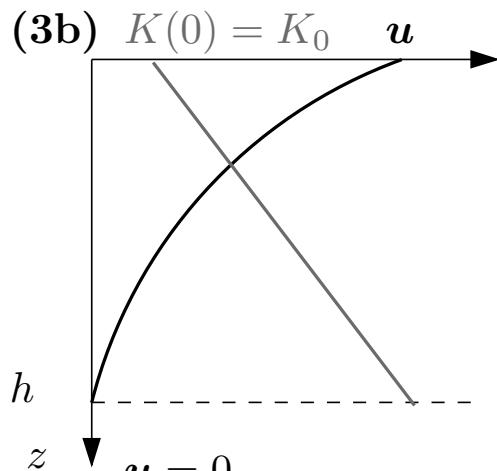


# theoretical transfer functions $H(z)$



$$H_\nu(z) = \frac{e^{-i\pi/4}}{\rho\sqrt{(2\pi\nu + f)K_0}} \frac{\sinh [(1+i)(h-z)/\delta_1]}{\cosh [(1+i)h/\delta_1]}$$

$$\delta_1 = \sqrt{\frac{2K_0}{2\pi\nu + f}}$$

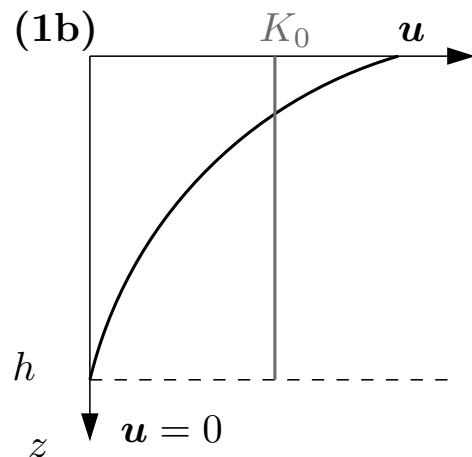


$$H(z) = \frac{1}{\rho\sqrt{i(2\pi\nu + f)K_0}} \times$$

$$\frac{\mathcal{I}_0 \left[ 2\sqrt{\frac{i(z_0 + h)}{\delta_2}} \right] \mathcal{K}_0 \left[ 2\sqrt{\frac{i(z_0 + z)}{\delta_2}} \right] - \mathcal{K}_0 \left[ 2\sqrt{\frac{i(z_0 + h)}{\delta_2}} \right] \mathcal{I}_0 \left[ 2\sqrt{\frac{i(z_0 + z)}{\delta_2}} \right]}{\mathcal{I}_1 \left[ 2\sqrt{\frac{iz_0}{\delta_2}} \right] \mathcal{K}_0 \left[ 2\sqrt{\frac{i(z_0 + h)}{\delta_2}} \right] + \mathcal{K}_1 \left[ 2\sqrt{\frac{iz_0}{\delta_2}} \right] \mathcal{I}_0 \left[ 2\sqrt{\frac{i(z_0 + h)}{\delta_2}} \right]}$$

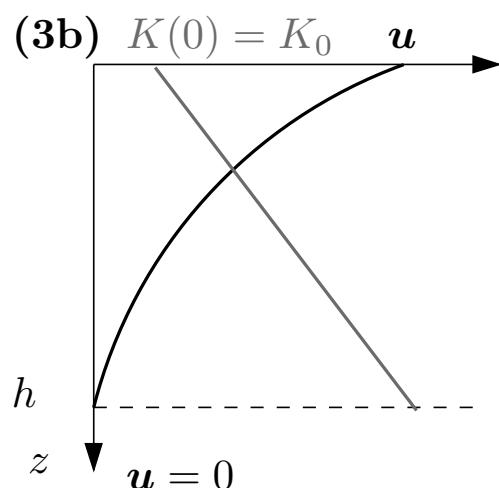
$$\delta_2 = \frac{K_1}{2\pi\nu + f} \quad z_0 = \frac{K_0}{K_1}$$

# theoretical transfer functions at the inertial frequency



$$\nu \rightarrow -f/2\pi, \quad \delta_1 \rightarrow +\infty$$

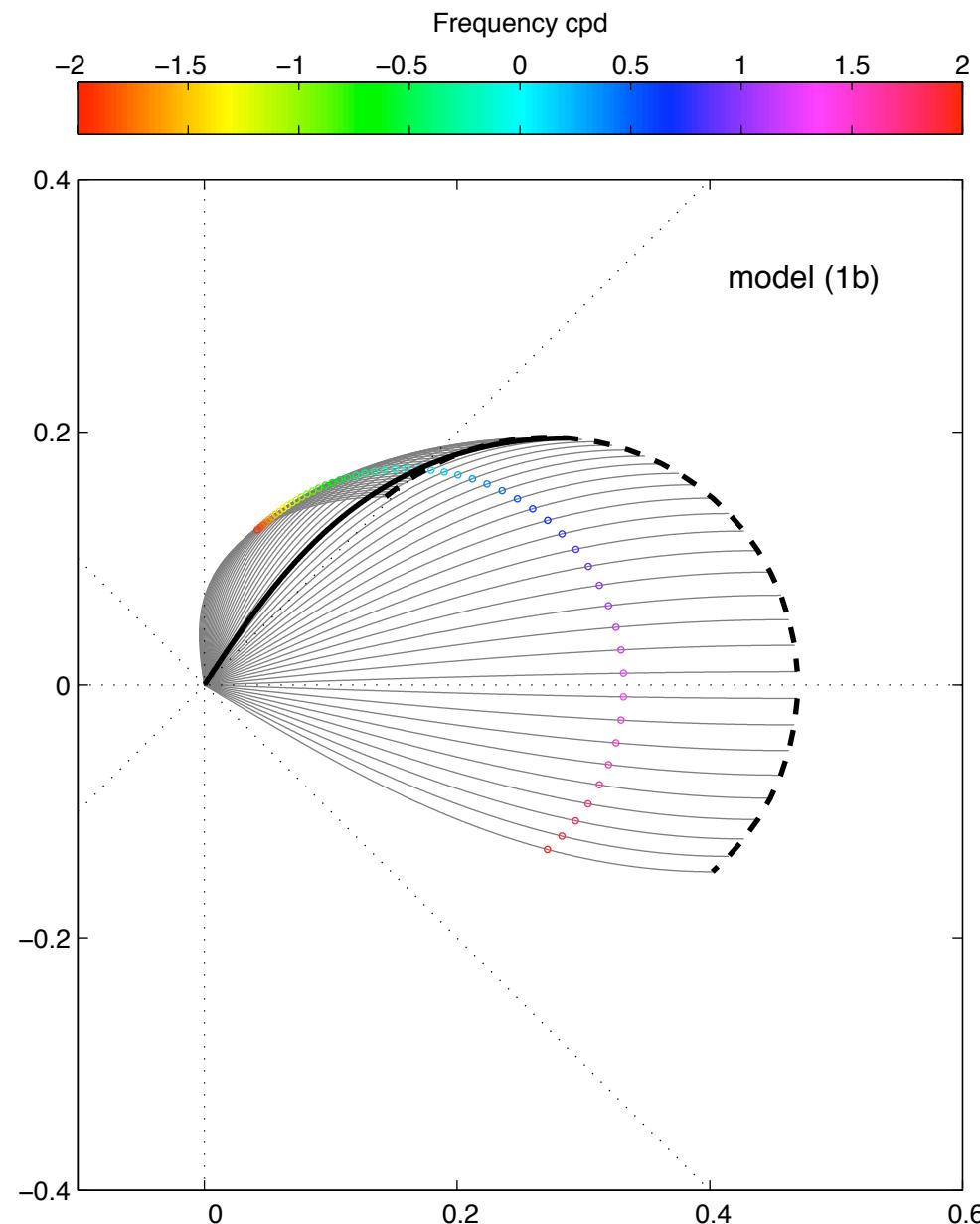
$$H \sim \frac{h-z}{\rho K_0}$$



$$\nu \rightarrow -f/2\pi, \quad \delta_2 \rightarrow +\infty$$

$$H \sim -\frac{1}{\rho K_1} \ln \left( \frac{z_0 + z}{z_0 + h} \right)$$

# theoretical transfer functions $H(z)$



$$\phi = -41$$

$$K(z) = K_0$$

$$K_0 = 0.1061 \text{ m}^2\text{s}^{-1}$$

$$h = 51 \text{ m}$$

$$-f = 1.3 \text{ cpd}$$

# theoretical transfer functions $H(z)$

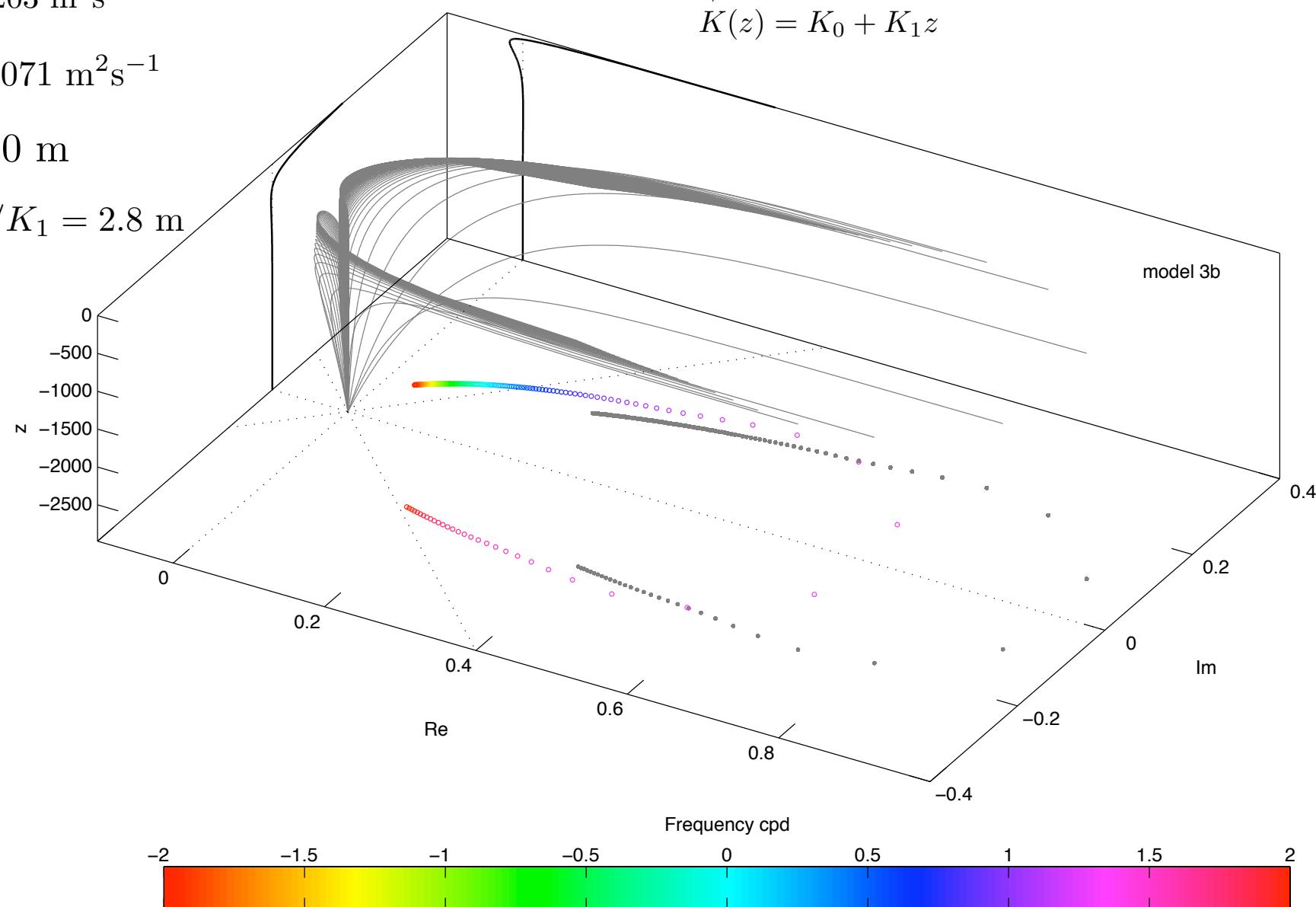
$$K_0 = 0.0203 \text{ m}^2\text{s}^{-1}$$

$$K_1 = 0.0071 \text{ m}^2\text{s}^{-1}$$

$$h = 2980 \text{ m}$$

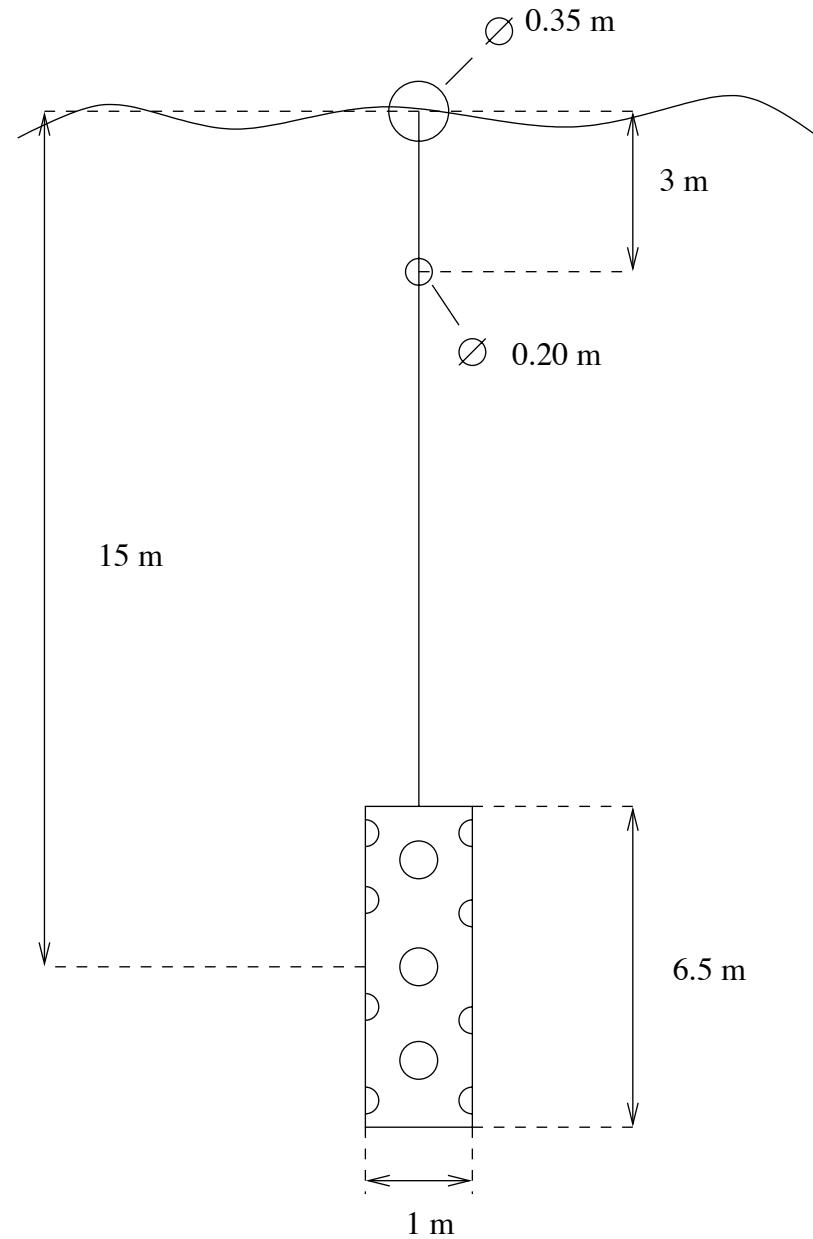
$$z_0 = K_0/K_1 = 2.8 \text{ m}$$

$$\phi = -41$$
$$K(z) = K_0 + K_1 z$$



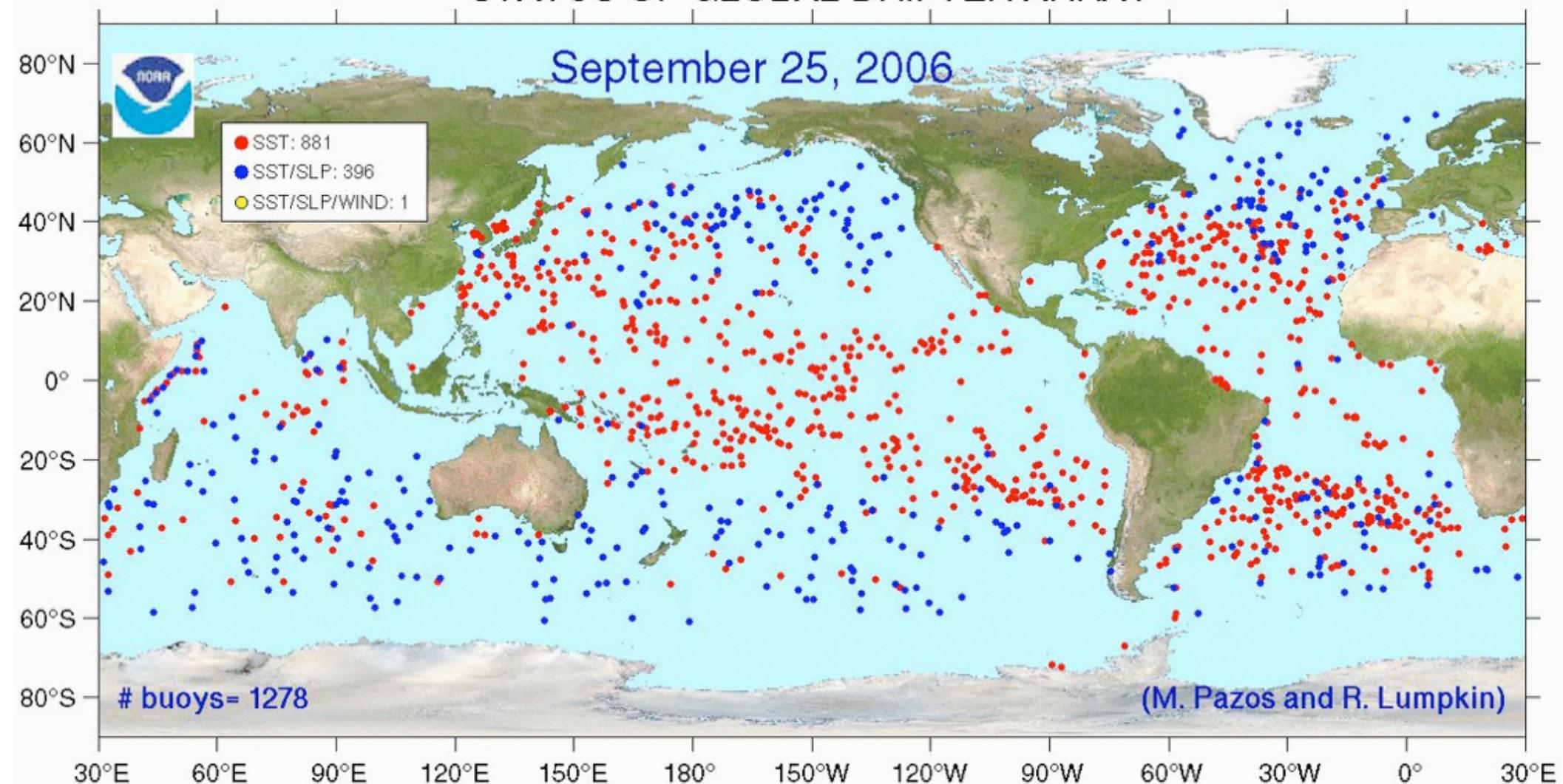
## 2. Estimates from surface drifter velocity data

# SVP drifter



# drifter data

## STATUS OF GLOBAL DRIFTER ARRAY



# drifter, altimeter and wind stress data

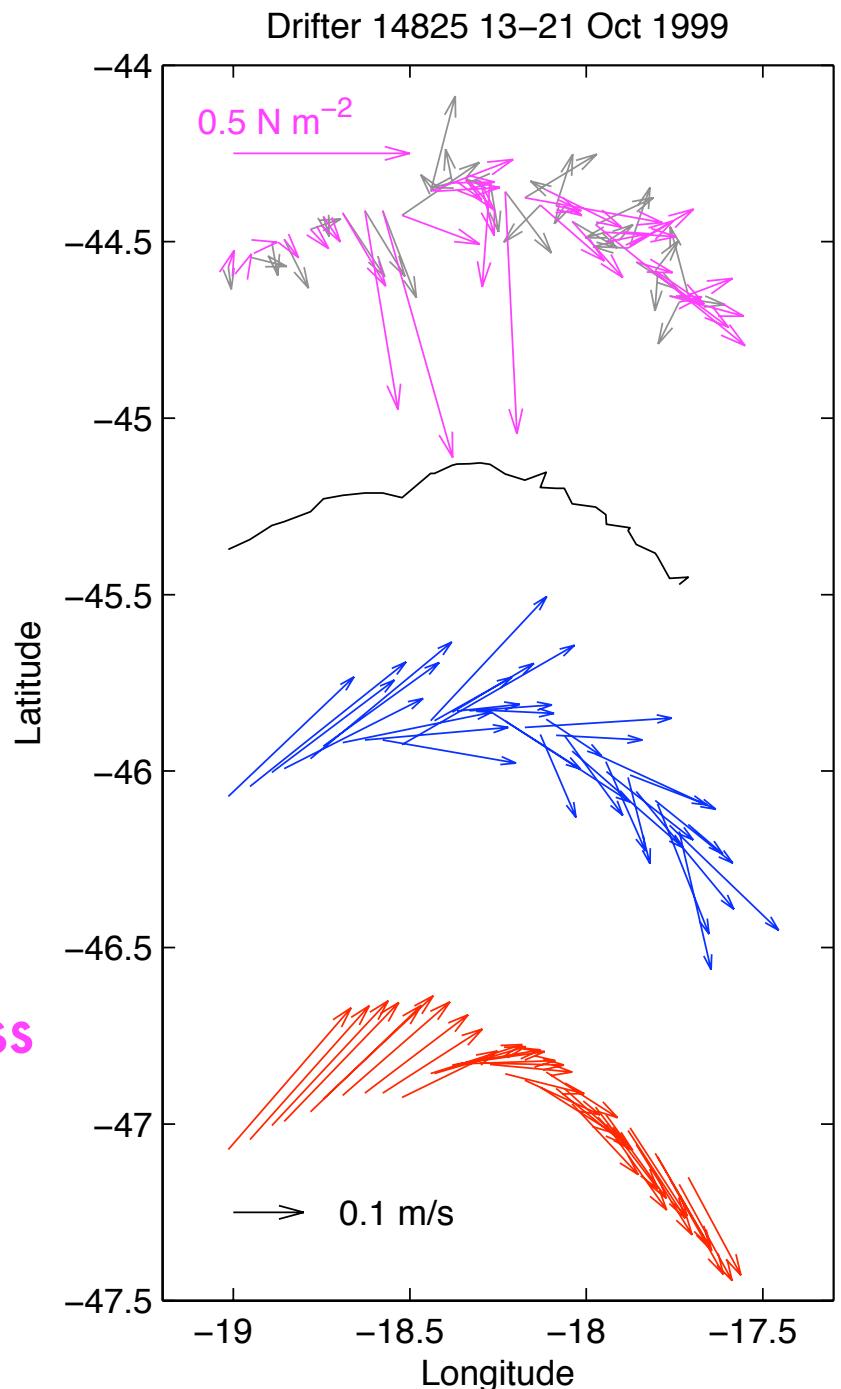
$$\mathbf{u}_{\text{drifter}}(t) = \mathbf{u}_{\text{geo}}(t) + \mathbf{u}_{\text{ageo}}(t)$$

$\tau(t)$  stress

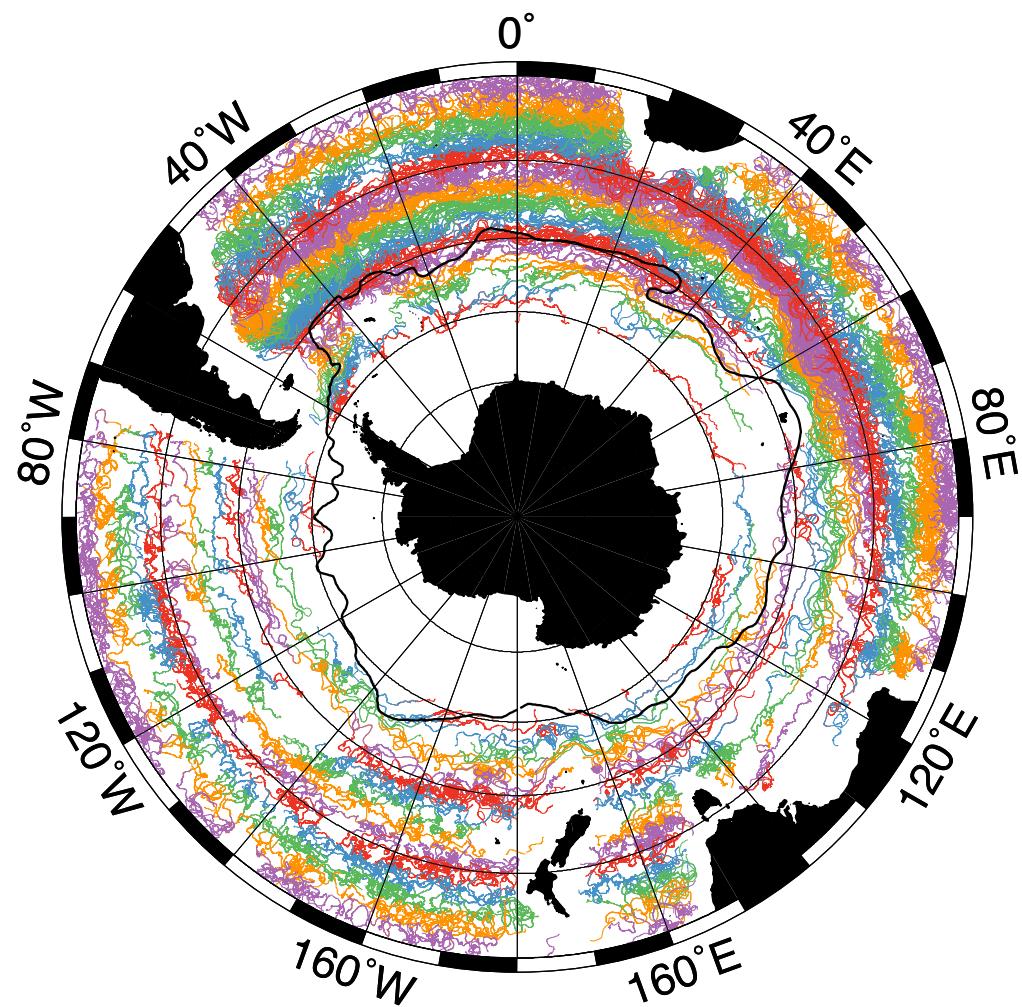
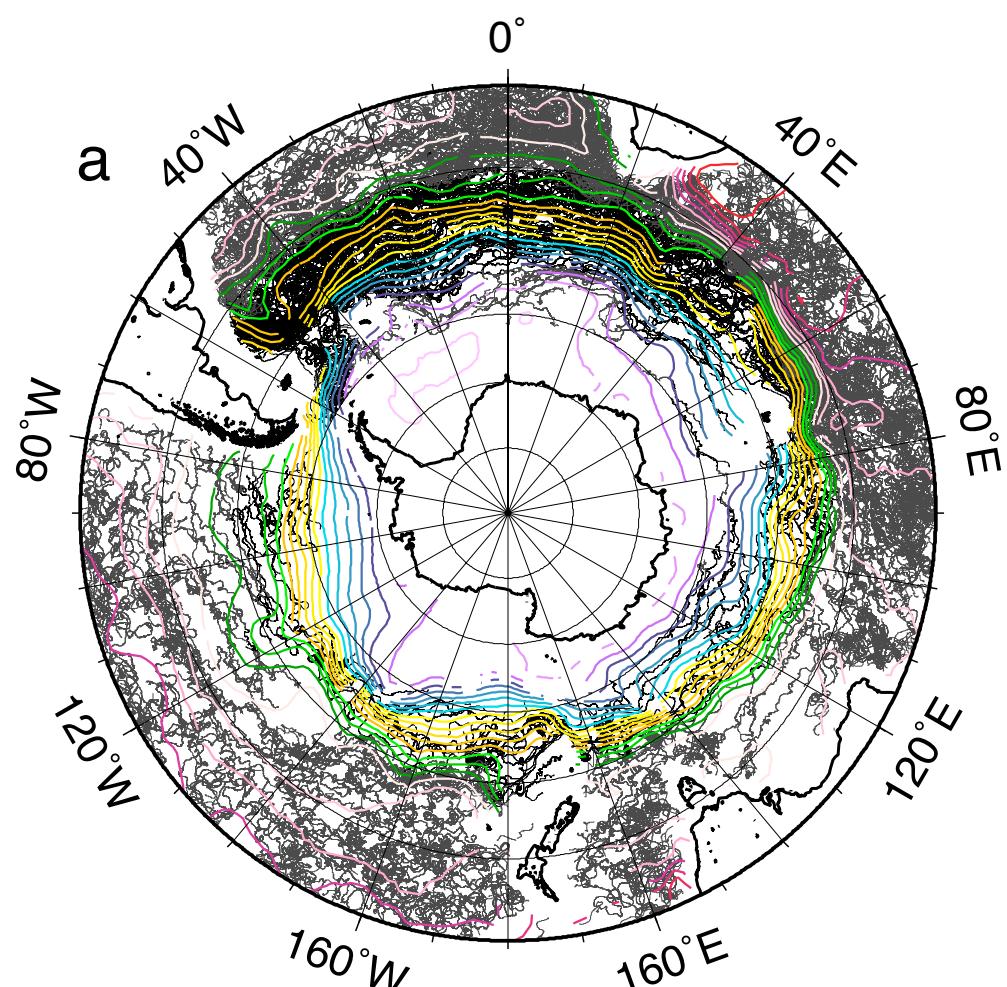
SVP 6-hourly data

AVISO altimeter ssh maps

ECMWF ERA-40 reanalyses wind stress



# data in the Southern Ocean

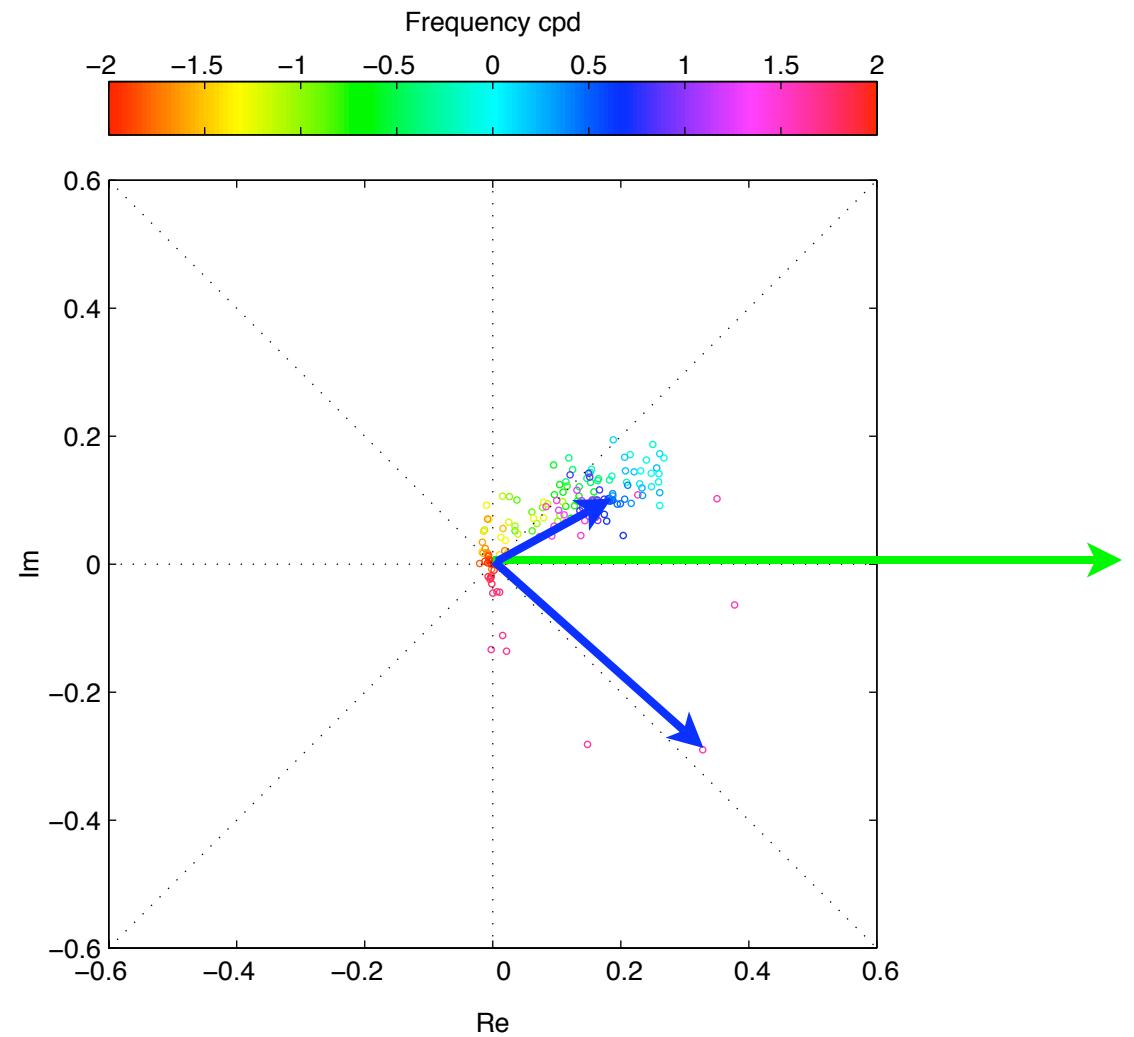


# Estimates of the transfer function

53°S

$$S_{\tau u}(\nu, z) = \mathbf{H}(\nu, z) S_{\tau \tau}(\nu)$$

$$\hat{\mathbf{H}}(\nu_k, z) = \frac{\hat{S}_{\tau u}(\nu, z)}{\hat{S}_{\tau \tau}(\nu)} = \frac{\langle \mathbf{T}_k \mathbf{U}_k^* \rangle}{\langle \mathbf{T}_k \mathbf{T}_k^* \rangle}$$



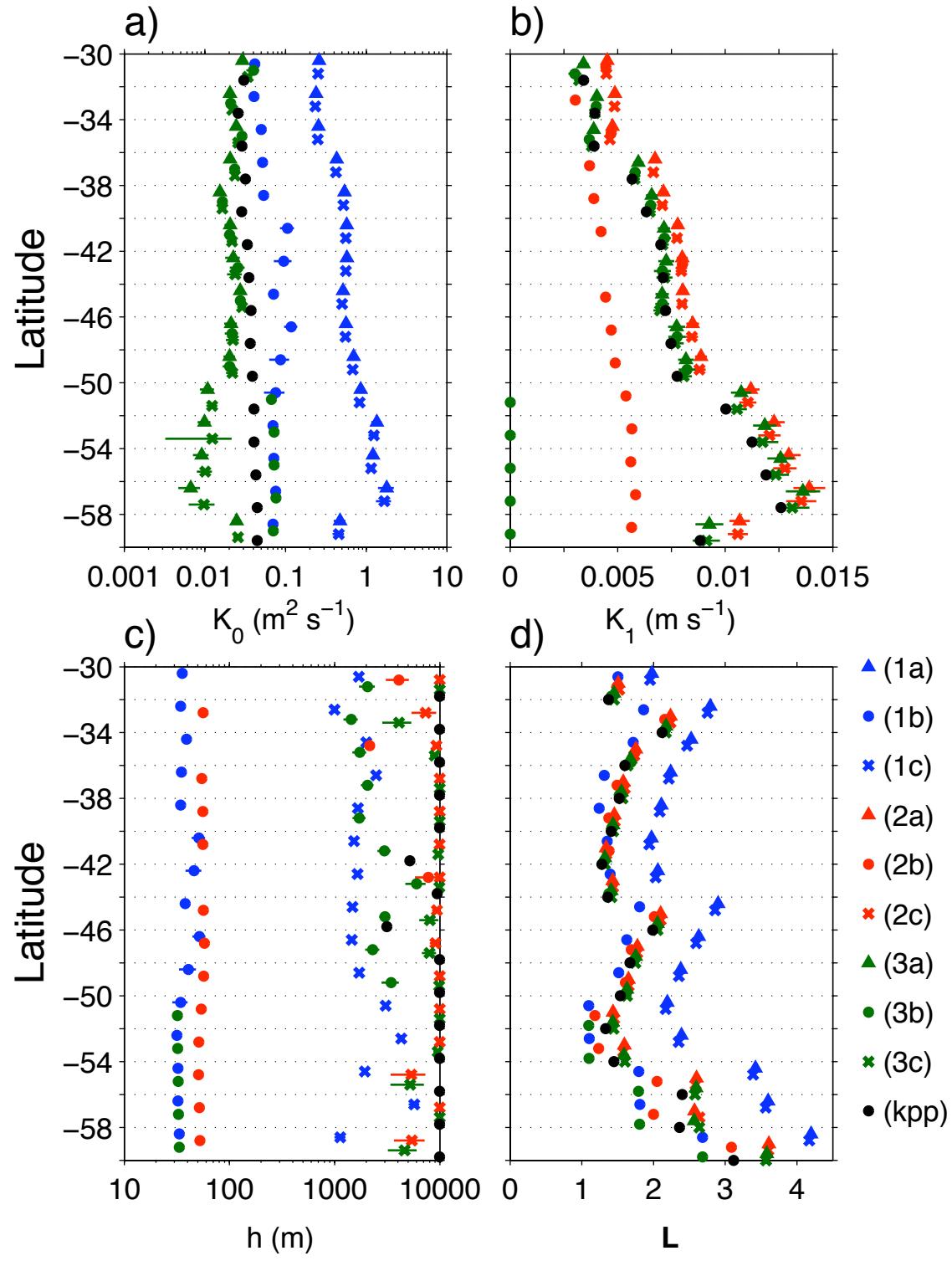
**minimization of a cost function:**

$$L = \sum_{\nu_k} |H_m(\nu_k, 15) - \hat{H}(\nu_k, 15)| \times \gamma^2(\nu_k)$$

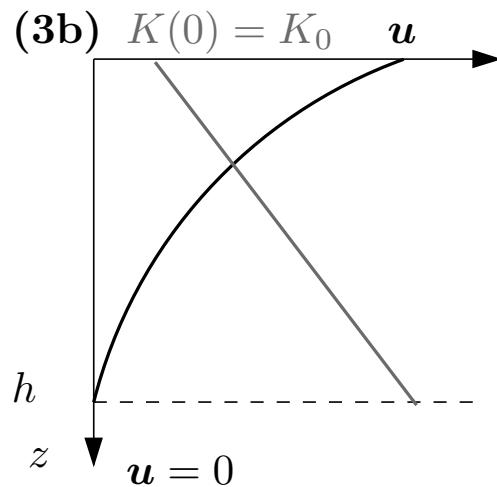
**parameter space:**  $K_x, h$

**uncertainties by bootstrapping methods**

### 3. Results in the Southern Ocean



# A zonal synthetic view of the Ekman layer in the SO



$$K_0 \approx 0.02 - 0.04 \text{ m}^2\text{s}^{-1}$$

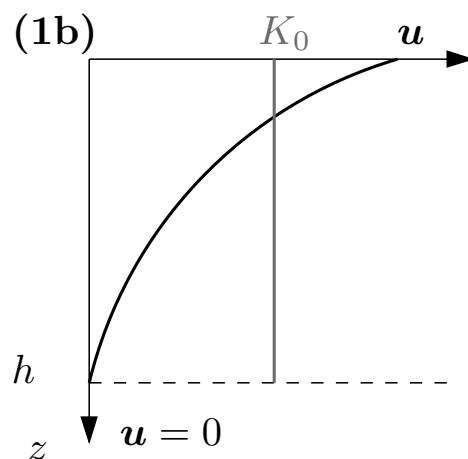
$$K_1 \approx 2 - 10 \times 10^{-3} \text{ m s}^{-1} \sim 0.55 u^*$$

$$z_0 \approx 2 - 13 \text{ m}$$

$$h = O(10^3) \text{ m}$$

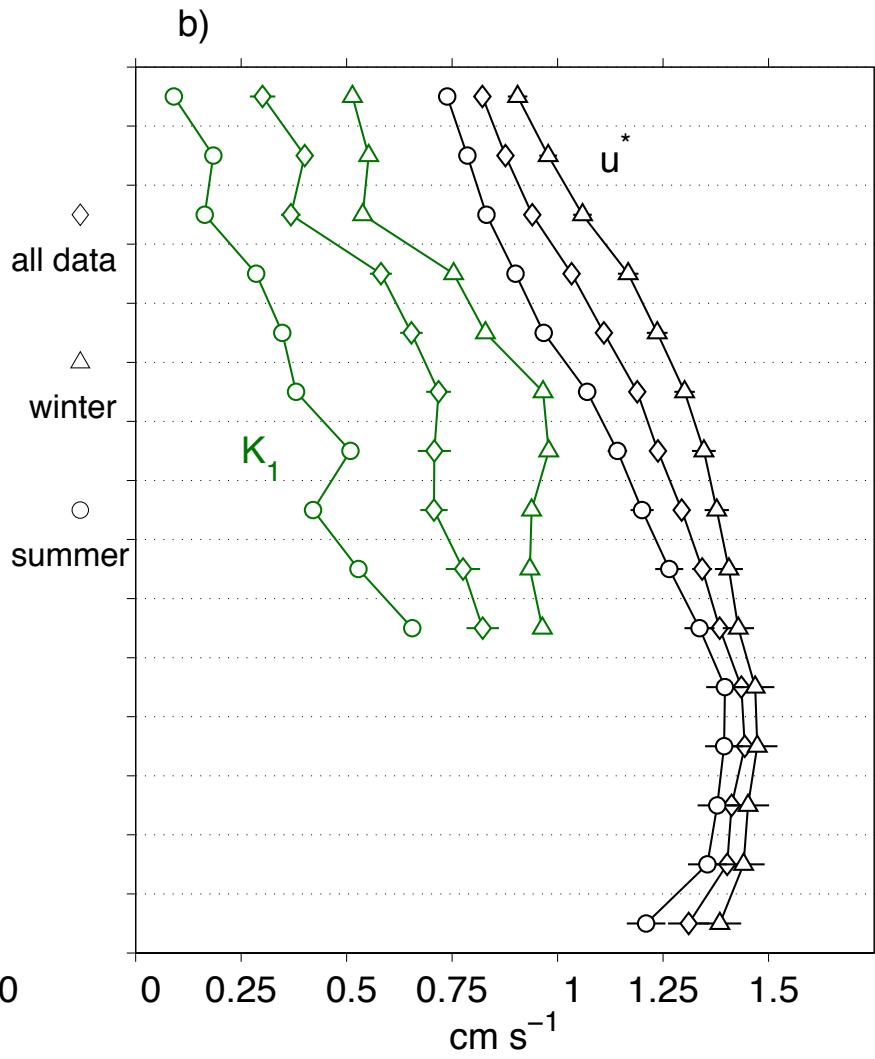
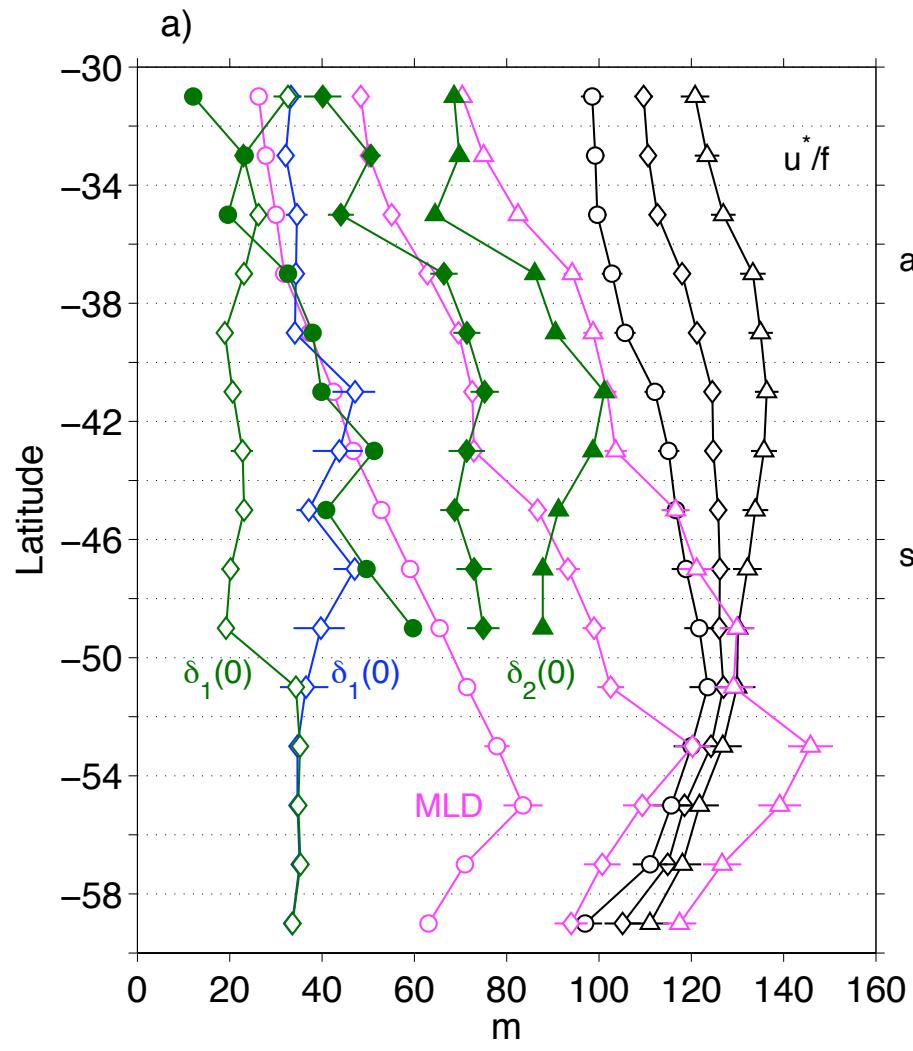
50 S

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$$K_0 \approx 0.07 \text{ m}^2\text{s}^{-1}$$

$$h = 30 - 40 \text{ m}$$



$$\delta_1 = \sqrt{\frac{2K_0}{2\pi\nu + f}}$$

$$\delta_2 = \frac{K_1}{2\pi\nu + f}$$

$$u^* = \sqrt{\frac{|\tau|}{\rho}}$$

MLD: Shenfu Dong, SIO

# Summary

- Ekman “spiral” depends on  $K(z)$  and the frequency of the forcing
- Drifter data can provide estimates of  $K$  on global scales
- The Ekman layer has different characteristics, north and south of 50 S, with relationships to the wind stress and the stratification